Scientifically-Based Mathematics Instruction at Tier 1

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Overview

There has been a recent firestorm of attention to learning outcomes in education. What caused the first spark is debatable and has been linked to a number of historical keystone events (e.g., Sputnik, civil rights movement), but several converging events eventually ignited a change in the way that educational services are delivered and evaluated by consumers. Criticism of so-called traditional methods of helping children who struggle to master important skills in schools via the special education system as lacking treatment utility (Hayes, Nelson, & Jarrett, 1987; Macmann & Barnett, 1999) or consequential validity (Messick, 1995) challenged the efficacy and ethical basis for special education service delivery to students diagnosed with learning disabilities. The publication of numerous policy statements and panel reports calling for the use of rapid identification of need and rapid response in ways that are known to improve the problem, and accountability for producing improvements (Donovan & Cross, 2002; NCTM, 2000; National Reading Panel, 2000; U.S. Department of Education, 2003) created an impetus for sensitive progress monitoring data to evaluate and troubleshoot learning outcomes. Legislation emphasizing student learning outcomes and scientific evidence as the path most likely to lead to improved learning outcomes (No Child Left Behind Act, 2001 and Individuals with Disabilities in Education, 2004) provided a framework for system reform spanning both general and special education. These sparks ignited a firestorm that has caused educators to re-think the very purpose of instruction and to reach pragmatic conclusions that instruction ought to produce measurable results and assessments ought to reflect the potential for instruction to produce results given the right conditions (Neisworth & Bagnato, 1992). The seminal work of curriculum-based measurement (CBM) researchers operating out of Minnesota provided a framework within which practitioners could produce meaningful and measurable improvements in child learning and moved the field from problem admiration to problem solving (Deno, 1986). This work was instrumental in establishing the foundation for Response to Intervention systems of decision making in schools. Response to intervention (RTI) rapidly became a vehicle for system reform because it provided a framework within which data could be used to make relative judgments (e.g., determining who needs help the most and how much they need) and to distribute instructional resources to promote the greatest good for the greatest number of students. RTI is the culmination of many decades of research and advocacy work and embodies the aspirations of policies and procedures that seek to improve student learning for all students as the first and foremost priority of public schooling.

Reading instruction and learning has undergone dramatic revision over the past decade as a result of system reform efforts focusing on early intervention, frequent progress monitoring, and evaluation of response to intervention in determining which students need which instructional resources to demonstrate success in reading. During this time, less attention has been paid to mathematics. For example, the ratio of studies available to guide the National Reading Panel to studies available to guide the National Mathematics Advisory Panel was about 6:1. However, mathematics has become a primary focus more recently in systems where reading achievement is on track but mathematics achievement is comparably lacking. Mathematics is a notable target for system change since many U.S. children fail to meet minimal proficiency standards before the end of their formal schooling (NAEP, 2005) and children who live in poverty or represent ethnic minorities are lower-achieving than their majority peers and grow at a slower pace (meaning the gap between those at risk and those not at risk grows across grade levels). Despite the many shortcomings of the current state of mathematics instruction including a lack of streamlined instructional tools and textbooks and challenges in training and retaining highly skilled mathematics teachers, much is known about how to deliver effective instruction in mathematics (National Mathematics Advisory Panel, 2008). This paper will focus on describing elements of scientifically based instruction for core instruction in mathematics.

A recent paper written by Slavin and Lake (2008) summarized the state of the evidence with regard to the effectiveness of mathematics programs in elementary schools. These researchers used rigorous inclusion criteria (e.g., randomized or matched design, use of standardized comprehensive mathematics assessments to evaluate outcomes, minimum treatment duration of 12 weeks). Eighty-seven studies met their inclusion criteria (256 were reviewed and excluded). Despite the rigorous inclusion criteria, significant limitations were apparent in the 87 studies that were included in the review, including an overreliance on post-hoc matched analyses (as opposed to prospective random assignment) and selection biases (operating on the front end in those who were selected or volunteered to participate and on the back-end via attrition). Of the 87 studies that were analyzed, 13 studies evaluated effects of various mathematics curricula with a median effect size of +0.10. This finding is a notable finding because most systems that are beginning to think about implementing multi-tiered mathematics instruction to enhance mathematics achievement schoolwide have no empirical basis for selecting a core curriculum that will return measurable effects for students. Stated another way, of the research studies that are available on this question, no curricula demonstrated even moderate effect sizes on student learning in mathematics. Thirty-six studies examined computer-assisted instructional programs that are used to supplement core instruction in mathematics. Unlike the other types of programs considered, the computer-assisted instructional programs emphasized computational skills over problem solving or conceptual understanding. The studies in this category were both more numerous (36 versus 13) and more rigorous. The median effect size for these studies was +0.19, nearly double that of the core curricula programs. Thirty six studies examined instructional process strategies (e.g., cooperative learning, mastery learning, direct instruction, and others). These studies included procedures designed to change the quality of teacher-student interactions in the classroom to advance student learning in math and represented the highest quality studies included in this review. The median effect size for instructional process strategies was +0.33.

Several conclusions may be reached from the Slavin and Lake analysis. First, as already mentioned, the evidence does not support one core curriculum over another. Second, supplemental instructional programs delivered on the computer can accelerate student learning, particularly on computational skills. *Third, if you really want to alter learning outcomes in mathematics, key consideration must be given to the quality of the instructional interaction that occurs in the classroom between teacher and student*. The purpose of this paper is to summarize key recommendations for delivering the highest-quality mathematics instruction possible.

Identify Essential Skills

***Begin with number sense*.** Because math is highly proceduralized and continually builds on previous knowledge for successful learning, early deficits have enduring and devastating effects on later learning, (e.g., National Mathematics Advisory Panel, 2008; National Council of Teachers of Mathematics [NCTM], 2000; U.S. Department of Education, 2003). A firm basis in number sense that begins during the toddler years and is established by third grade is fundamental to later mathematics development (Ginsburg, Lee, and Boyd, 2008). Number sense has been defined by Griffin (2004) as a child knowing that numbers represent quantity and therefore have magnitude, that one number may be bigger or more than another number (or quantity), and that numbers occupy fixed positions in a counting sequence and therefore have a fixed order with numbers appearing later in the sequence representing more (greater quantities) than numbers appearing earlier in the sequence (Griffin, 2004). Hence number sense underlies early skills like counting in sequence, counting objects, comparing quantities and determining which quantity is greater, and adding and subtracting numbers. These skills pave the way for the development of more sophisticated mathematical understanding like an understanding of place value, associative property, commutative property, and distributive property.

Integrate procedural and operational instruction with applied practice and conceptual understanding. Seminal research in mathematics suggests that understanding of procedures and operations emerges through repeated exposure to and practice solving computations. Conceptual understanding emerges in concert with and as a result of these experiences solving problems in a bi-directional rather than linear relationship. For example, as children become more fluent in early addition, most children naturally progress from counting two sets of objects to attain a sum to using what has been called the “count on” strategy (starting with the larger number and counting up from that number to add the smaller number to the larger number and obtain a sum) which is more efficient. The “count on” strategy reflects a child’s ability to rapidly compare two quantities or numbers, conclude which number is larger, and count in sequence beginning from a fixed position on a number line (belying an understanding of ordinality). Repeated experiences answering addition problems confirms and solidifies emerging understanding of these concepts while deeper understanding of the concepts enhances more competent problem solution in the future (Baroody, 1985; Baroody, 1999). For example, as children gain competence they may solve 5 + 4 by adding 5 + 5 and subtracting 1 to arrive at the correct sum, both demonstrating and advancing an understanding of associative property (an understanding that the order in which the operations are performed does not affect the resulting quantity so given $a + b – c$ the solution can be obtained by adding *a* and *b* and then subtracting *c* or by subtracting *c* from *b* before adding a without affecting the solution). To accomplish this key objective in mathematics instruction teachers should (1) provide adequate opportunity for students to practice key computation skills to fluency, and (2) provide systematic guidance to develop conceptual understanding of major principles in mathematics including building time into the instructional program for students to make estimations, discuss possible solutions, verify solutions, and alter procedures to solve slightly different or more complex problems over time (Fuchs et al., 2002; Fuchs et al., 2003).

Sequence Skills in a Logical Way. Because mathematics skills build in a predictable way, curriculum and instruction leaders in LEAs and SEAs should take care to sequence the introduction of skills in manageable "chunks," provide sufficient time for students to master skills that are introduced, and check frequently to ensure that students have retained previously mastered skills. Logical sequences of computational and operational skills can be developed that follow state-specified standards for mathematics learning at each grade level. Instructional leaders at the SEAs and LEAs should emphasize a streamlined instructional plan in mathematics that allows for depth of understanding over time rather than trying to cover too much information without sufficient depth. Such streamlining will require tough decisions about which skills to prioritize and the National Math Panel Report (2008) provides guidance about what skills merit priority during mathematics instruction (see pp. 15-22 of the National Math Panel Report, 2008).

Provide Adequate Instructional Time .If students are to learn the skills that are introduced, then sufficient high-quality instructional time must be provided. Once a logical sequence of skills that are expected to be learned during the academic year at each grade level has been specified, leaders must work backward to ensure that adequate instructional time and resources are provided to attain that goal. All too often, skills are introduced without sufficient time and opportunity for students to master the skill or concept. The instructional program then moves on to a new skill. It is counterproductive to narrowly introduce a skill and move on before the skill has been mastered. Hence, developing a calendar of instruction for mathematics that provides a sequence within which skills will be introduced and includes verification that the skills have been mastered before moving on to new content is critical to attaining desired outcomes. The National Math Panel recommends streamlining instruction to focus on key objectives that progress in logical difficulty from preK to grade 8 that provides students the skills needed to be successful in algebra (National Math Panel, 2008). The panel report is a direct, appropriate, and timely indictment of historically typical classroom practices where operations and concepts are introduced without sufficient depth and to no meaningful conclusion. According to the National Math Panel, skills critical to establishing a foundation for successful learning of algebra by grade 8 include fluency with whole numbers (addition and subtraction by grade 3, multiplication and division by grade 5) and fluency with fractions (including representation of fractions, decimals, and percentages; operations with fractions, decimals, and percentages; operations with positive and negative integers; operations with positive and negative fractions; and solving percentages, ratios, and rates to balance equations) (see p. 20, Table 2 of the National Math Panel Report, 2008 for listing of benchmark skills). Critically, to attain these benchmarks, students must demonstrate a consistently strong trajectory in mastering basic computation, more advanced computation, and a rapidly more sophisticated understanding of important concepts like equivalence, place value, and scale. From a leadership perspective, improving mathematics competence in an SEA or LEA requires prioritizing mathematics instruction, providing adequate time for mathematics instruction each day (50 minutes is not likely to be sufficient in most systems), and monitoring growth toward identified benchmarks to ensure that improvement efforts are having the desired effect on student outcomes in mathematics.

Routinely Measure Student Learning. Data support the utility of computation measures for reaching screening decisions. Key decisions that must be made at screening include: Is there a system-wide, grade-wide, or class-wide learning problem in mathematics? Where grade-wide and system-wide problems exist, Tier 1 solutions must include evaluating the adequacy of instructional efforts provided through Tier 1. If system-level problems are ruled out, then the next decision is whether individual student learning problems exist and how those learning problems can most efficiently be addressed. Screening data pay large dividends to systems by providing a structured opportunity to articulate expected skills and their progression across grade levels, to evaluate learning relative to these stated benchmarks, and to re-align instruction to expectations for learning to enhance effects. The screening data become a basis to evaluate a system's progress over time at addressing learning deficits (or stated differently, instructional problems) that are identified in the system. With ongoing progress monitoring, any system or instructional change becomes a hypothesis to be tested and so long as adults interpret and respond to the data collected, a problem solution will likely be attained for most identified problems. Table 1 summarizes suggested screening probes in mathematics for grades Pre-K through grade 8.

Instructional Strategies must be selected based on Individual Student Skill Competence. Understanding the Instructional Hierarchy (IH; Haring, Lovitt, Eaton & Hansen, 1978) is central to advancing student learning. The Instructional Hierarchy reflects the dynamic interface between learner capacity and instructional strategies. According to the Instructional Hierarchy, students progress through the following stages of learning in linear sequence: acquisition, proficiency, and generalization. Identifying the stage of learning where the student is functioning matters because that information links directly to particular instructional strategies that will accelerate learning when applied. At the acquisition stage of learning, the skill is not yet established. Student responses may be hesitant and are often incorrect. Effective instructional strategies for skills at this stage of learning include modeling correct responding, providing prompts and cues to guide the student to correctly complete the skill, and interrupting incorrect responses to provide immediate corrective feedback. Feedback may be somewhat elaborate at this stage with the goal being to help the learner understand exactly the conditions under which a response is correct or incorrect. The student has completed the acquisition stage of learning when the response is accurate without adult assistance. The proficiency stage of learning is focused on building fluency. At this stage of learning the response is correct and independent, but not automatic. In the proficiency stage of learning, effective instructional strategies include uninterrupted periods of independent practice with reinforcement for more fluent (e.g., more accurate and rapid, Binder, 1996) performance. Goal setting and self-monitoring strategies are effective tools to accelerate learning at this stage. When the student response has become fluent, the proficiency stage of learning is complete and the student can move into the generalization stage. At the generalization stage of learning, the student learns to respond correctly under slightly different task demands (slightly different conditions, settings, problem presentations). Providing students the opportunity to practice responding in different settings, different contexts, with slightly variable tasks will be the most effective strategies at this stage. Many of the strategies that were helpful at the previous two stages will be helpful at this stage too, including, for example, embedding cues or prompts to respond correctly, providing corrective feedback to guide, and then reinforce correct generalization, and then providing more extended intervals to practice recognizing the need for generalization and correctly generalizing the newly mastered skill.

Not only does the IH indicate which strategies are likely to be effective based on student competence, but also indicates strategies that will be ineffective. Knowing that a student's skill is at the acquisition stage indicates that the student is not ready for independent work at this level. Independent work for skills at the acquisition stage is likely to result in response errors that go uncorrected and become more intractable and has been associated with disruptive student behavior as demonstrated in the seminal study conducted by Gickling & Armstrong (1978). Similarly, corrective feedback (generally a powerful instructional strategy) can actually interfere with practice time and decrease the number of opportunities to respond that an individual student may experience. When the response is correct but slow (as when the student enters the proficiency stage of learning), the goal is to allow brief intervals of uninterrupted practice so that the child experiences as many opportunities to respond as possible during the practice interval.

The IH is the science that ought to drive instructional activity in the classroom each day. If IH is the science that drives instruction, then two implications are evident: student performance must be monitored at routine intervals and adults must be responsive to the data. Deno and Mirkin (1977) characterized effective instruction as instruction that is responsive to the child. More recently, Olson, Daly, Andersen, Turner, & LeClair, (2007) echoed this statement, asserting that adults are the ones who must be responsive for RTI systems to work, responsive to the data, that is.

Address System Barriers to Excellent Tier 1 Instruction

Excellent implementation efforts occur where there is a plan around which the instructional leaders who are responsible for the change have consensus and the will to implement fully (VanDerHeyden & Witt, 2007). A notable finding highlighted earlier in this paper is the finding that the largest effect sizes for learning in mathematics occurred where the instructional environment was altered to permit a better alignment between instructional strategy and child competence (e.g., instructional match through the instructional hierarchy), instructional time was maximized, child opportunities to respond were maximized, a system was in place to monitor student learning and provide the student feedback on his or her performance (Slavin & Lake, 2008). Variables that are essential to instructional excellence in mathematics include:

- Access to sufficiently sequenced materials that permit children the opportunity to practice skills to fluency in a logical order that progresses as students gain competence and periodically reviews previously mastered skills.
- Prioritized or essential standards or expectations for student learning in mathematics at each grade level.
- Adequate time allocated to mathematics instruction.
- High-quality instructional interactions in the classroom where instructional strategies are aligned to student skill competence, corrective feedback is provided, and students are actively engaged throughout the lesson.
- Adequate opportunities to practice skills to high levels of proficiency.
- Instruction focused on applying mastery-level computational skills to solve more complex or novel problems. Guided classroom discussion to estimate or predict, identify possible solutions, and evaluate solution accuracy.

- Progress monitoring system to evaluate the degree to which classes, grades, schools, and finally individual children are meeting expected benchmarks for learning during the course of instruction.
- Adults respond to student level data to troubleshoot instructional effects, beginning first with integrity of the instructional environment (verifying each of the above conditions) and then systematically altering instruction while tracking the effects on student learning (e.g., reducing task difficulty, embedding incentives for accuracy or fluency, providing guided practice completing applied tasks).

Concluding Remarks on Integrity within RTI Systems

The effects of an RTI implementation depend upon correct selection and implementation of a series of interventions. Research has conclusively demonstrated that integrity of intervention implementation in classrooms tends to be poor even when excellent support is provided for implementation. Hence, RTI is particularly vulnerable to decision errors resulting from inadequate intervention implementation. An intervention or instructional plan that is not implemented consistently and correctly is a qualitatively different intervention that affects the technical adequacy of the RTI judgment. To illustrate this point, a case example is provided below. Following collection of screening data that indicated a system-wide learning problem in mathematics, a district decided to implement a supplemental intervention daily following a standard protocol to build fluency in key computational and procedural skills arranged in a logical sequence that matched the state standards. Percentage of correctly completed steps of the intervention for one teacher was poor (average integrity score across all sessions was 78%) and for the comparison teacher was strong (average integrity score across all sessions was 100%). The teacher with poor integrity overall averaged 59% correct completion of the intervention prior to beginning performance feedback and 96% following performance feedback. Hence, performance feedback successfully corrected the integrity problem that was present in that classroom. However, the class spent many weeks not progressing on the early skills while the integrity problem and this effect was not benign. Because the classroom had an integrity problem, valuable intervention time and effects were lost and were not recoverable preventing students in that class from experiencing the full range of skills targeted through intervention before the end of the school year (hence, students in each classroom experienced topographically and functionally different interventions). Because integrity of intervention implementation is positively correlated with student learning gains, untreated integrity problems often become student learning problems over time. Both novice and veteran implementation sites require consistent monitoring of integrity of instructional manipulations occurring at all tiers of the RTI effort to ensure that meaningful data can be obtained and accurate decisions can be based upon those data.

Implications for Teacher Preparation

RTI represents a fundamental shift in educational philosophy and policy. With a distinctively grassroots history, RTI is about promoting and advancing effective educational services. RTI policies and practices straddle the research to practice divide by embracing empiricism as a basis for decision making in the schools and demanding that science provide consumable findings that can be implemented with predictable effects. Hence, an important goal of all teacher preparation programs should be ensuring that teachers are prepared to be consumers of the evidence base for their profession. Teachers must be armed with the pragmatic know-how to collect progress-monitoring data and adjust their teaching strategies systematically for the greatest gain. Teachers should begin with the assumption that all children can learn given the right instruction and view themselves as the detective who will figure out how to best accelerate the learning of all the children in their care. As this shift occurs in systems, teachers' roles are shifting. The responsibility for all children's learning becomes the responsibility of all the adults in a system. Hence, the teacher need not function as an isolated teacher, but rather as part of a team where frequent datapoints are shared so that all teachers can bring their best efforts to help all children learn. Use of RTI demands new skills from teachers including (1) an understanding of screening and formative assessment, (2) an understanding of the principles of effective instruction as the basis for developing intervention strategies, (3) the ability to examine student data and reach conclusions about learning, (4) the ability to communicate with colleagues about problem-solving, and (5) the ability to access and use materials to deliver instruction in the most effective way possible. Importantly, many training programs may need to reconsider the degree to which their program accomplishes the objective of arming teachers with the pragmatic skills needed to be effective teachers.

There are also preparation challenges specific to mathematics teachers.

1. ***Effective teaching in mathematics improves individual student learning outcomes.*** Numerous reports have found that effective teaching has a so-called value-added effect on student learning with empirical findings demonstrating that successive placements into classrooms with less effective teachers was associated with lower achievement scores than successive placement into more effective teachers' classrooms (National Math Panel, 2008; U.S. Department of Education, 2003).

2. ***Effective mathematics teachers are difficult to find, to hire, and to retain.*** All recent policy reports have found that mathematics teachers typically do not demonstrate the content knowledge needed to teach mathematics (Ginsburg, Lee, & Boyd, 2008; National Math Panel, 2008; National Research Council, 2001; U.S. Department of Education, 2003). Math teachers at all grade levels scored 568 on the mathematics section of the SAT, a score that is below the national average and well below the 624 score obtained by math majors who are not interested in teaching (US Department of Education, 2003). Most math majors do not choose to go into teaching and most math teachers were not math majors in college (US Department of Education, 2003). The National Research Council report *Adding it Up* (2001) found that most math teachers fail to demonstrate a sufficient grasp of mathematics beyond simple calculations, citing seminal research studies in the field. Critically, mathematics teachers have difficulty explaining the conceptual basis for mathematical operations, like the place-value concepts underlying multi-digit multiplication or regrouping with subtraction (Ma, 1999). Teachers must be fluent not only with the procedures required for successful problem solving but also with the conceptual bases for those procedures so that they can successfully guide students (at varying ability levels) to make the connections between the computations they perform and the functional meaning of those computations. Teachers with higher competencies in mathematics tend to leave the profession at

higher rates and earn much higher salaries when working in professions outside of teaching (U.S. Department of Education, 2003). Efforts to correct the problem of teachers being adequately prepared in the content knowledge needed to teach mathematics must be broad including efforts to attract individuals to the teaching profession with a particular interest in mathematics, establishing higher professional standards for certification to teach mathematics, providing monetary incentives for initial recruitment and retention of effective teachers, and taking corrective actions to enhance the capacity of teachers currently in the workforce who are currently responsible for teaching mathematics. On the last point, Ginsburg et al. note that typical workshops that lack sufficient depth or assume too much knowledge will not be effective. These authors call for pragmatism in providing teachers with a scientific framework for what must be taught and why, and then arming teachers with the tools needed to do so effectively in their own classrooms. The implication here is that such professional development will be sustained, formative (tied to learner effects and adjusted as needed), and will use performance coaching to ensure that the desired skills are practiced to fluency. The *Adding it Up* report advocates for five characteristics of effective mathematics teaching including conceptual understanding of the core knowledge required in the practice of teaching, fluency in using basic instructional routines, strategic competence in planning effective instruction an troubleshooting problems as they arise, adaptive reasoning in explaining instructional techniques selected and reflecting on their utility, and productive disposition toward mathematics, teaching, learning, and professional development. Hence, effective teaching in mathematics requires content expertise, instructional expertise, and the motivation and will to deliver it.

3. ***Math instructional resources and tools are cumbersome, disjointed, and ineffective.*** Jitendra and colleagues have conducted a series of studies examining the extent to which textbooks of mathematics met specified instructional design criteria with generally dismal findings. In research spanning from 1996 to 2005, Jitendra et al. found that few textbooks met basic instructional design criteria

including providing clear objectives, explicit teaching explanations, sufficient examples of correct and incorrect responding, and effective feedback. To the degree that the textbook sets the occasion for these instructional activities, teaching effects are compromised. Given that the quality of instructional interaction between the child, the teacher, and the learning materials is highly related to child learning outcomes (Slavin & Lake, 2008), urgent reform is needed to provide better instructional tools for teachers of mathematics. All recent panels on mathematics instruction have called for (1) consensus about essential outcomes of mathematics instruction across the grades, (2) streamlining of content across the grade levels, (3) teaching to mastery using a variety of instructional strategies that are matched to the needs of the learner and range from explicit instruction in computation to accelerated programs for advanced learners who demonstrate rapid mastery of learning objectives, and (4) use of formative assessment to guide instruction in mathematics (NCTM, 2000; National Math Panel, 2008; National Research Council, 2001).

4. ***Embrace empiricism through research and individual problem-solving.*** As noted earlier in this paper, there are limited data to draw on in reaching recommendations about effective core instructional programs in mathematics. Such data are likely to be complicated by a variety of factors including for example the settings in which the programs are implemented and the quality with which the programs are implemented. Policies and the contingencies they bring (through accountability) are often tantamount to improved practices and resource development and deployment. All major policy panels have explicitly noted the need for high-quality research to inform policy recommendations for mathematics instruction and there is consensus that the recommendations should be specific and ready for implementation. As research proceeds, systems and teachers can immediately implement recommended practices that focus on improving the quality of the student-teacher interactions, can advocate for identifying essential learning outcomes, sequencing instructional tasks in a logical and distributed way that allows sufficient time for mastery, integrating computation with conceptual

level instruction, monitoring progress of learners and adjusting instruction to keep learning on track, and making a system commitment to emphasize empiricism over pedagogy.

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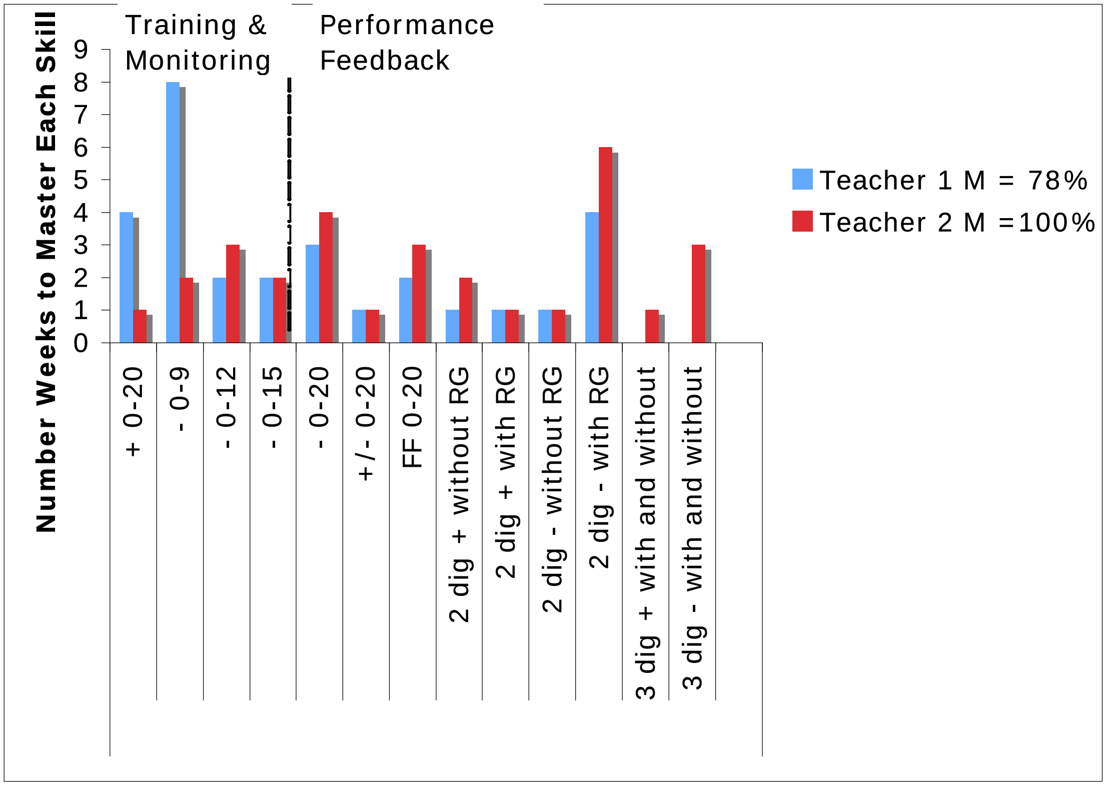
Table 1

Suggested Screening Probes for Grades Pre-K through 8

	Screening Fall	**Screening Spring**	**Progress Monitoring**
Pre-K	Counting Objects Aloud; Select a Number (1-10); Rapid Discrimination	Counting Objects Aloud; Rapid Number Naming	
Kindergarten	Counting Objects and Selecting Matching Number (1-10); Quantity Discrimination; Rapid Discrimination	Counting Objects and Writing Number (1-10)	
1st Grade	Sums to 5	Sums to 18 or 20	Addition and Subtraction 0-20
2nd Grade	Addition and Subtraction 0-20	Multi-digit addition or subtraction without regrouping	Fact Families Addition/Subtraction 0-20
3rd Grade	Fact Families Addition/Subtraction 0-20 or 3-digit addition and subtraction with and without regrouping (this is hard for most third graders but reflects a skill that most are expected to be able to do)	Multiplication 0-9 or 0-12	Multiplication and Division 0-12

4th Grade	Fact Families Multiply/Divide 0-12	Multi-digit multiplication without or with regrouping	Multi-digit division with and without remainders
5th Grade	Multi-digit multiplication with and without regrouping	1 digit into 2-3 digit dividend with remainders	Reduce fractions
6th Grade	Decimals multiplication	Find least common denominator	Substitution of whole number to solve equations
7th Grade	Mixed operations for integers	Mixed operations for fractions or percentages	Substitution of fraction to solve equations
8th Grade	Mixed operations for fractions	Solve simple algebraic proportions	Solve percentages (e.g., x% of 10 = 5 and 50% of x = 10)

Figure 1


Training &
Monitoring
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Number Weeks to Master Each Skill
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3
2
1
0
+ 0-20
- 0-9
- 0-12
- 0-15
- 0-20
+/- 0-20
FF 0-20
2 dig + without RG
2 dig + with RG
2 dig - without RG
2 dig - with RG
3 dig + with and without
3 dig - with and without
Teacher 1 M = 78%
Teacher 2 M =100%